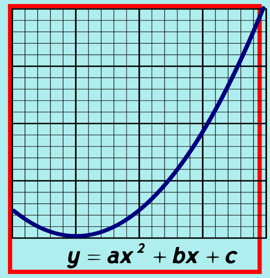
Math 25

Fall 2017

Lecture 10



Class Quiz

① find the first 4 terms of

$\{2n+1\}_{n=1}^{4}$

$n=1 \quad a_1 = 2(1)+1 = 3$

$n=2 \quad a_2 = 2(2)+1 = 5$

$n=3 \quad a_3 = 2(3)+1 = 7$

$n=4 \quad a_4 = 2(4)+1 = 9$

3, 5, 7, 9

② find the sum: $\sum_{n=1}^{4}(n^2-1)$

$= \underset{n=1}{(1^2-1)} + \underset{n=2}{(2^2-1)} + \underset{n=3}{(3^2-1)} + \underset{n=4}{(4^2-1)}$

$= 0 + 3 + 8 + 15 = \boxed{26}$

Ch. 8

Factorials

$$n! = n(n-1)(n-2)(n-3)\cdots 3\cdot 2\cdot 1$$

$$6! = 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1 = \boxed{720}$$

$$\frac{12!}{5!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot \cancel{5!}}{\cancel{5!}}$$

$$= 12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6 = \boxed{}$$

By Def.: $0! = 1$

Simplify: $\frac{8!}{5!\cdot 3!} = \frac{8\cdot 7\cdot \cancel{6}\cdot \cancel{5!}}{\cancel{5!}\cdot \cancel{3\cdot 2\cdot 1}}$

$$= 8\cdot 7 = \boxed{56}$$

Simplify: $\frac{(2n+1)!}{(2n)!} = \frac{(2n+1)\cdot \cancel{(2n)!}}{\cancel{(2n)!}} = 2n+1$

$a_n = \frac{(n+2)!}{n!}$ Find the first 4 terms.

$a_1 = \frac{3!}{1!} = \frac{6}{1} = \boxed{6}$ $a_2 = \frac{4!}{2!} = \frac{4\cdot 3\cdot \cancel{2!}}{\cancel{2!}} = \boxed{12}$

$a_3 = \frac{5!}{3!} = \frac{5\cdot 4\cdot \cancel{3!}}{\cancel{3!}} = \boxed{20}$ $a_4 = \frac{6!}{4!} = \frac{6\cdot 5\cdot 4!}{4!} = \boxed{30}$

$$a_n = \frac{(n+3)!}{(n-1)!}$$ Find a_1 and a_{10}.

$$a_1 = \frac{(1+3)!}{(1-1)!} = \frac{4!}{\underbrace{0!}_{\text{By Def.}}} = \frac{24}{1} = \boxed{24}$$

$$a_{10} = \frac{(10+3)!}{(10-1)!} = \frac{13!}{9!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot \cancel{9!}}{\cancel{9!}}$$

$$= \boxed{17160}$$

Find the first 5 terms of the sequence defined by $a_n = 4a_{n-1} - 1$, $a_1 = 5$

$a_1 = \boxed{5}$

$a_2 = 4a_1 - 1 = 4(5) - 1 = \boxed{19}$

$a_3 = 4a_2 - 1 = 4(19) - 1 = \boxed{75}$

$a_4 = 4a_3 - 1 = 4(75) - 1 = \boxed{299}$

$a_5 = 4a_4 - 1 = 4(299) - 1 = \boxed{1195}$

Given $a_n = 2n + 5$

① Write the first 4 terms.

$\boxed{7, 9, 11, 13}$

② find $a_{50} = 2(50) + 5 = \boxed{105}$

③ Is this an arithmetic sequence? if yes Give a_1 & d

Yes, common difference

$a_1 = 7$, $d = 2$

④ find S_{50}

$S_n = \frac{n}{2}[a_1 + a_n]$ $S_{50} = \frac{50}{2}[7 + 105]$

$= 25(112)$

$= \boxed{2800}$

Consider

3, 10, 17, - - - - -, 437

1) Is this an arithmetic sequence? Yes

2) find a_1 & d. $a_1 = 3$, $d = 10 - 3 = 7$

$d = 17 - 10 = 7$

$\boxed{d = 7}$

3) find a general formula for the $\underbrace{\text{nth term}}_{a_n}$.

$a_n = a_1 + (n-1)d$

$= 3 + (n-1) \cdot 7 = 3 + 7n - 7 = \boxed{7n - 4}$

$a_n = 7n - 4$

4) find a_{20}

$a_n = 7n - 4$

$a_{20} = 7(20) - 4 = \boxed{136}$

5) what term is 437?

$a_n = 437$

$7n - 4 = 437 \rightarrow 7n = 441$

$n = \frac{441}{7}$ $\boxed{n = 63}$

$a_{63} = 437$

6) find S_{100}

$S_n = \frac{n}{2}[a_1 + a_n]$

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_{100} = \frac{100}{2}[2 \cdot 3 + (100-1)7]$

$= 50[6 + 700 - 7]$

$= 50[699]$

$\boxed{S_{100} = 34950}$

Find the Sum: $\sum_{n=1}^{40} (2 - 5n)$

$a_1 = 2 - 5(1) = -3$

$a_2 = 2 - 5(2) = -8$ $\Rightarrow$

$a_3 = 2 - 5(3) = -13$

Arithmetic sequence

$a_1 = -3$, $d = -5$

Now we need to find S_{40}

$S_{40} = \frac{\overset{n}{40}}{2}[2(\overset{a_1}{-3}) + (\overset{n}{40} - 1) \cdot (\overset{d}{-5})] = 20[-6 - 200 + 5]$

$= 20[-201] = \boxed{-4020}$

How long does it take to pay off a loan of \$3960 if we pay \$50 first month, \$60 second month, \$70 third month, and so on.

$$50, 60, 70, \cdots\cdots$$

$$\underbrace{50 + 60 + 70 + \cdots\cdots}_{\text{How many payments?}} = 3960$$

Arithmetic Sequence $a_1 = 50$, $d = 10$

find n where $S_n = 3960$

$$S_n = \frac{n}{2}\left[a_1 + a_n\right]$$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$3960 = \frac{n}{2}\left[2 \cdot 50 + (n-1) \cdot 10\right]$$

$$3960 = \frac{n}{2}\left[100 + 10n - 10\right]$$

$$3960 = \frac{n}{\cancel{2}}\left[\overset{45}{\cancel{90}} + \overset{5}{\cancel{10}}n\right]$$

$$3960 = n(5n + 45)$$

$\underbrace{3960 = n(5n+45)}$ → Divisible by 5

$792 = n(n+9)$

$n^2+9n-792=0$

using Quadratic formula

$a=1 \quad b=9 \quad c=-792$

$b^2-4ac = 9^2-4(1)(-792) = 3249$

$$n = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-9 \pm \sqrt{3249}}{2} = \frac{-9 \pm 57}{2}$$

$$n = \frac{-9+57}{2} = \frac{48}{2} = \boxed{24}$$

$$\cancel{n = \frac{-9-57}{2} = -\#}$$

Not applicable

24 monthly Payments.

Given 5, 10, 20, ...

① write the next 4 terms

5, 10, 20, $\boxed{40, 80, 160, 320}$ ...

② what kind of Sequence? Arithmetic or $\boxed{\text{Geometric}}$

$a_1 = 5$, Common ratio $\Rightarrow r=2$

a_1 First Term

r Common ratio

a_n nth term

$a_n = a_1 \cdot r^{n-1}$

S_n Sum of first n term.

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

③ find a_{10} $a_{10} = a_1 r^{10-1}$

$= 5 \cdot 2^9 \Rightarrow \boxed{a_{10} = 2560}$

④ find S_{10} $S_{10} = \dfrac{a_1(1-r^{10})}{1-r}$

$S_{10} = \dfrac{5(1-2^{10})}{1-2}$ $\boxed{S_{10} = 5115}$

Consider a geometric sequence with

$a_1 = 4$, $a_2 = 12$

1) find common ratio r.

$r = \dfrac{a_2}{a_1} = \boxed{3}$

2) find a_6 $a_6 = a_1 \cdot r^{6-1}$

$= 4 \cdot 3^5 = \boxed{972}$

3) find S_8

$S_8 = \dfrac{a_1(1-r^8)}{1-r} = \dfrac{4(1-3^8)}{1-3} = -2(1-3^8)$

$= \boxed{13120}$

Consider a geometric sequence with

$a_1 = -15$, $a_4 = \frac{-5}{9}$

1) find the common ratio r.

we know $a_n = a_1 \cdot r^{n-1}$

$a_4 = a_1 \cdot r^3$

$\frac{-5}{9} = -15 r^3$

$r^3 = \frac{-5/9}{-15}$

$r^3 = \frac{1}{27}$

$\boxed{r = \frac{1}{3}}$

2) find a_6

$a_6 = a_1 \cdot r^5$

$= -15\left(\frac{1}{3}\right)^5$

$\boxed{a_6 = -\frac{5}{81}}$

3) find S_7

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_7 = \frac{-15\left(1-\left(\frac{1}{3}\right)^7\right)}{1-\frac{1}{3}} = \frac{-15\left(1-\frac{1}{3^7}\right)}{\frac{2}{3}}$

$= \frac{-45}{2}\left(1-\frac{1}{3^7}\right)$

$= \boxed{\frac{-5465}{243}}$

Consider the sequence

$a_1, a_1 r, a_1 r^2, a_1 r^3, \ldots$

If add the terms

$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \ldots$

Infinite Geometric Series

If $|r| < 1$, then the sum of the terms $S = \frac{a_1}{1-r}$

If $|r| \geq 1$, there is no sum.

Find the sum

$1000 + 500 + 250 + \ldots$

$S_\infty = S = \frac{a_1}{1-r}$

$a_1 = 1000$

$r = \frac{500}{1000} = \frac{1}{2}$

$r = \frac{250}{500} = \frac{1}{2}$

$r = \frac{1}{2}$

$S = \frac{1000}{1 - \frac{1}{2}}$

$= \frac{1000}{1/2} = \boxed{2000}$

Find $.\overline{3}$ in fraction.

$.\overline{3} = .333333\ldots$

$= .3 + .03 + .003 + .0003 + \ldots$

$a_1 = .3 \quad r = \frac{.03}{.3} = \frac{.03}{.30} = \frac{3}{30} = \frac{1}{10}$

$S = \frac{a_1}{1-r} = \frac{.3}{1-\frac{1}{10}} = \frac{.3}{\frac{9}{10}} = .3 \cdot \frac{10}{9}$

$= \frac{3}{9} = \frac{1}{3}$

$\boxed{.\overline{3} = \frac{1}{3}}$

Write $1.8\overline{25}$ as a reduced fraction

$1.8\overline{25} = 1.825252525252525\ldots$

$= 1.8 + \underbrace{.025 + .00025 + .0000025 + \ldots}$

$a_1 = .025$

$r = \frac{.00025}{.025} = \frac{.00025}{.02500}$

$= \frac{25}{2500} = \frac{1}{100}$

$S = \frac{.025}{1-\frac{1}{100}} = \frac{.025}{\frac{99}{100}} = \frac{2.5}{99}$

$= \frac{25}{990} = \frac{5}{198}$

$= 1.8 + \frac{5}{198}$

$= \frac{18}{10} + \frac{5}{198}$

$= \frac{1807}{990}$

write $.7\overline{8}$ in reduced fraction.

$.7\overline{8} = .788888\ldots$

$= .7 + .088888888\ldots$

$= .7 + .08 + .008 + .0008 + .00008 + \ldots$

$= .7 + \dfrac{.08}{1 - \frac{1}{10}}$

$= \dfrac{7}{10} + \dfrac{.08}{\frac{9}{10}} = \dfrac{7}{10} + \dfrac{.8}{9} = \dfrac{7}{10} + \dfrac{8}{90}$

$= \boxed{\dfrac{71}{90}}$ $\boxed{\dfrac{71}{10} = .7\overline{8}}$

Mathematical Induction

working with mathemical statement

P_n, n is a positive integer.

To show that it works

1) we verify that P_1 works

2) we assume that P_K works,

then we show that P_{K+1} works.

Prove by mathematical induction:

$2 + 6 + 10 + \cdots + (4n-2) = 2n^2$

$P_1 \to n=1$ LHS RHS

$2 = 2(1)^2$ ✓

$P_2 \to n=2$ $2+6 = 2(2)^2$ ✓

$P_3 \to n=3$ $2+6+10 = 2(3)^2$ ✓

Assume P_K works

$2 + 6 + 10 + \cdots + (4k-2) = 2k^2$

Add the next term

$2 + 6 + 10 + \cdots + (4k-2) + (4k+2) = 2k^2 + 4k+2$

$\underbrace{\overbrace{2 + 6 + 10 + \cdots + (4k-2)}^{k \text{ terms}} + (4k+2)}_{\#\text{ of terms}} = 2k^2 + 4k + 2$

k+1 terms

$= 2(k^2 + 2k + 1)$

$= 2(k+1)^2$

Prove by mathematical induction:

$1 + 3 + 3^2 + 3^3 + \cdots + 3^{n-1} = \frac{1}{2}(3^n - 1)$

$a_n = 3^{n-1}$

$a_1 = 3^{1-1} = 3^0 = 1$ $a_3 = 3^{3-1} = 3^2$

$a_2 = 3^{2-1} = 3^1 = 3$ $a_4 = 3^{4-1} = 3^3$

Check P_1 LHS RHS

$P_1 \to n=1$ $1 = \frac{1}{2}(3^1 - 1)$ ✓

$P_2 \to n=2$ $1+3 = \frac{1}{2}(3^2-1)$ ✓

Assume it works for P_K

$$1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1} = \frac{1}{2}(3^k - 1)$$

Add the next term of pattern to both sides

$$1 + 3 + 3^2 + 3^3 + \cdots + 3^{k-1} + 3^k = \frac{1}{2}(3^k - 1) + 3^k$$

$$= \frac{1}{2}(3^k - 1) + \frac{2}{2} \cdot 3^k = \frac{1}{2}(3^k - 1) + \frac{1}{2} \cdot 2 \cdot 3^k$$

$$= \frac{1}{2}\left[3^k - 1 + 2 \cdot 3^k\right] = \frac{1}{2}\left[3 \cdot 3^k - 1\right]$$

Final Ans

$$= \frac{1}{2}(3^{k+1} - 1)$$

Show that $5^n - 1$ is divisible by 4.

this means $5^n - 1 = \text{Some number} \cdot 4$

6 is divisible by 2 $\Rightarrow 6 = 3 \cdot 2$

96 " " " 4 $\Rightarrow 96 = 24 \cdot 4$

So we need to show $5^n - 1 = N \cdot 4$

$n = 1 \qquad 5^1 - 1 = 5 - 1 = 4 = 1 \cdot 4$ ✓

$n = 2 \qquad 5^2 - 1 = 25 - 1 = 24 = 6 \cdot 4$ ✓

$n = 3 \qquad 5^3 - 1 = 125 - 1 = 124 = 31 \cdot 4$ ✓

assume it works for $n=k$

$$5^k - 1 = N \cdot 4$$

we need to show now that

$$5^{k+1} - 1 = M \cdot 4$$

$$5 \cdot 5^k - 1 = (4+1)5^k - 1$$

$$= 4 \cdot 5^k + 1 \cdot 5^k - 1$$

$$= 4 \cdot 5^k + \boxed{5^k - 1}$$

$$= 4 \cdot 5^k + M \cdot 4$$

$$= (5^k + M) \cdot 4$$

So $5^n - 1$ is divisible by 4.

$$5^{k+1} - 1 = \# \cdot 4$$

Binomial Thrm

$(a+b)^n$

$$(a+b)^0 = 1$$

$$(a+b)^1 = a + b$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$\vdots$$

If we work with Coefs.

Pascal Triangle

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

$$(a+b)^6 = a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + \cdots$$

Binomial Coef. $\binom{n}{r} = \dfrac{n!}{r! \cdot (n-r)!}$

In Your calc.

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

find ${}_5C_2 = \binom{5}{2} = \dfrac{5!}{2! \cdot (5-2)!} = \dfrac{5!}{2 \cdot 3!}$

$= \boxed{10}$

find $\binom{8}{0} = {}_8C_0 = \dfrac{8!}{0! \cdot (8-0)!} = \boxed{1}$

Binomial Thrm

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2$$

$$+ \binom{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n} a^0 b^n$$

find the first 3 terms of $(a+b)^8$ expansion

$$(a+b)^8 = \binom{8}{0} a^8 b^0 + \binom{8}{1} a^7 b^1 + \binom{8}{2} a^6 b^2$$

$$= 1a^8 + 8a^7 b + 28a^6 b^2$$

find the 6th term of expansion of $(a+b)^{14}$.

$$\binom{14}{5} a^9 b^5$$

$$= 2002\, a^9 b^5$$

find the tenth term of

$$(y^3 + 2z^2)^{14}$$

find 10th term of $(a+b)^{14}$

$$\binom{14}{9} a^5 b^9 \Rightarrow 2002 (y^3)^5 (2z^2)^9$$

$$= 2002 \cdot y^{15} \cdot 2^9 \cdot z^{18}$$

$$= \boxed{1025024\, y^{15} z^{18}}$$

For $|x| < 1$

$$(1+x)^n = 1 + nx + \frac{n(n-1)x^2}{2!} + \frac{n(n-1)(n-2)x^3}{3!} + \cdots$$

$$(1.5)^3 = \curvearrowright 3.375$$

$$(1 + .5)^3 = 1 + 3(.5) + \frac{3(3-1)(.5)^2}{2!} + \frac{3(3-1)(3-2)(.5)^3}{3!} + \cdots$$

$$= 1 + 1.5 + 3(.5)^2 + 1\cdot(.5)^3 + \cdots$$

$$= 1 + 1.5 + .75 + .125 \approx 3.375$$